

Dividing Fractions and Mixed Numbers - Black Problems

1. While driving, Ron averages r miles per hour for d miles and then averages $2r$ miles per hour for d miles. Overall, Ron's average speed equals kr miles per hour. What is the value of k ? Express your answer as a common fraction.

2. Evaluate $\frac{\frac{4}{5} + \frac{1}{3}}{\frac{4}{5} - \frac{1}{3}}$

3. James and Sophie are selling pens on a charity day. All pens are the same price. James and Sophie start the day with an equal number of pens. At the end of the day, James has sold three-quarters of his pens, and Sophie has sold four-fifths of hers. The difference in the money collected by each is \$18.

What was the combined amount of money that James and Sophie collected?

4. Wai Ling has spent two-fifths of her pocket money on food, and three-eighths of *the remainder* on a chocolate bar. Then she had \$1.50 left.

How much pocket money did she have to start with?

5. Simplify:

a. (i) $1 + \frac{1}{1+1}$

(ii) $1 + \frac{1}{1 + \frac{1}{1+1}}$

(iii) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1+1}}}$

(iv) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1+1}}}}$

b. (i) $1 + \frac{2}{3+4}$

(ii) $1 + \frac{2}{3 + \frac{4}{5+6}}$

(iii) $1 + \frac{2}{3 + \frac{4}{5 + \frac{6}{7+8}}}$

(iv) $1 + \frac{2}{3 + \frac{4}{5 + \frac{6}{7 + \frac{8}{9+10}}}}$

6. My answer is $\frac{3}{4}$. What is my question?

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

You should recall that to multiply two fractions you multiply numerator by numerator and denominator by denominator. For instance,

$$\frac{2}{5} \cdot \frac{3}{7} = \frac{2 \cdot 3}{5 \cdot 7} = \frac{6}{35}.$$

Dividing fractions is done by multiplying the first fraction by the reciprocal of the second. For instance,

$$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \cdot \frac{7}{3} = \frac{2 \cdot 7}{5 \cdot 3} = \frac{14}{15}.$$

In this section you will do the same thing with algebraic fractions (that is, with rational algebraic expressions).

Objective

Given two rational expressions, multiply them or divide them, and simplify the answer.

Cover the answer as you work these examples.

EXAMPLE 1

Given $\frac{12x}{15}$ and $\frac{20x}{18}$

$$\begin{aligned} \text{a. } & \frac{12x}{15} \cdot \frac{20x}{18} \\ &= \frac{12 \cdot 20x}{15 \cdot 18} \\ &= \frac{8}{9}x^2 \end{aligned}$$

$$\begin{aligned} \text{b. } & \frac{12x}{15} \div \frac{20x}{18} \\ &= \frac{12x}{15} \cdot \frac{18}{20x} \\ &= \frac{12 \cdot 18x}{15 \cdot 20x} = \frac{18}{25} \end{aligned}$$

Think These Reasons

Multiply the given expressions.

Multiply numerator by numerator and denominator by denominator.

Do the canceling.

Divide the first by the second.

Change dividing to multiplying by the reciprocal.

Multiply.

Do the canceling.

EXAMPLE 2

Divide and simplify: $\frac{x^2 - 2x - 15}{4x^2 + 8x} \div \frac{x + 3}{4x - 20}$

$$\frac{x^2 - 2x - 15}{4x^2 + 8x} \div \frac{x + 3}{4x - 20}$$

$$\frac{x^2 - 2x - 15}{4x^2 + 8x} \cdot \frac{4x - 20}{x + 3}$$

$$\frac{(x - 5)(x + 3)}{4x(x + 2)} \cdot \frac{4(x - 5)}{x + 3}$$

$$\frac{(x - 5)(x - 5)}{x(x + 2)}$$

$$\frac{x^2 - 10x + 25}{x^2 + 2x}$$

Write the given expression.

Write dividing as multiplying by the reciprocal.

Factor.

Multiply.

Cancel.

Multiply.

Simplify

7. $\frac{a^4 + ab^2}{a^3 - 2a^2b} \div \frac{a^3 + b^2}{(a - 2b)^2}$

8. $\frac{6x + 12}{8x - 4} \cdot \frac{3x - 9}{12x + 6} \div \frac{3x^2 - 12}{8x^2 - 2}$

9. $\frac{5a^2 - 5a}{4a^3} \cdot \frac{a^2 - 9a - 10}{4a - 40} \div \frac{2a^2 - 2}{a}$

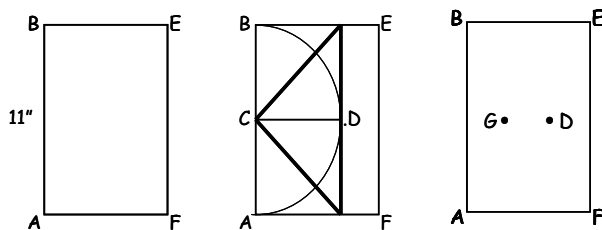
10. $\frac{x^3 - 9x}{8} \div \frac{x + 3}{2} \cdot \frac{4}{3x - 9}$

11. $\frac{b^2 - 7b + 10}{b^2 - 6b + 5} \div \frac{1}{b - 2} \cdot \frac{b + 1}{b^2 - 4b + 4}$

12. $\frac{t^2 + 2t - 8}{t + 4} \cdot (t - 2)^{-1}$

13. $\frac{2x + 3}{4x + 5} \div \frac{4x + 5}{2x + 3} \cdot \frac{8x + 10}{2x + 3}$

14. Find x if $\frac{x+3}{x-2} = 5$
15. For what value of x is $\frac{x-7}{x+2}$
- equal to zero?
 - undefined?
16. Peter painted $\frac{1}{3}$ of a room while Richard painted $\frac{2}{5}$ of the same room. It then took Peter 1 hour, 40 minutes to finish painting the remainder of the room by himself. In how many hours could Peter have painted the entire room by himself? Express your answer as a mixed number.
17. The sum of the reciprocals of three prime numbers is $\frac{167}{285}$. What is the sum of the three prime numbers?
18. Points A, B, E and F are the corners of an $8\frac{1}{2}$ -inch by 11-inch sheet of paper. Corners A and B are folded as shown below so that they meet at point D. The paper is then unfolded. In the same manner, corners E and F meet at point G. What is the length of segment GD? Express your answer as a mixed number.



Dividing Fractions and Mixed Numbers - Black Solutions

1. There are at least two good approaches to this problem that we recommended in the answer key above. The first method for solving the problem is to use a formula.

The equation that will get us to our answer is $\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$

In order to find the total time, we need to add the times it took for each part of the trip. Remember that time can be found by dividing the distance by the rate of speed. So by plugging in the values from the problem we find that...

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d + d}{\frac{d}{r} + \frac{d}{2r}} = \frac{2d}{\frac{3d}{2r}} = \frac{4}{3}r.$$

$$\text{Therefore } k = \frac{4}{3}.$$

The next method of solving uses the idea of creating a simpler case. Since the problem must work for any choice of d and r , select "nice" numbers. For example, let's let $d = 40$ miles and $r = 20$ miles per hour. We still need to know how an average speed is found, but we won't need to work with all of the

variables.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{40 + 40}{1 + 2} = \frac{80}{3}$$

$$\text{Therefore, } \frac{80}{3} = kr; \frac{80}{3} = 20k; k = \frac{4}{3}.$$

2. $2\frac{3}{7}$

3. \$558

4. \$4

5. a. (i) $1\frac{1}{2}$ (ii) $1\frac{2}{3}$ (iii) $1\frac{3}{5}$ (iv) $1\frac{5}{8}$

b. (i) $1\frac{2}{7}$ (ii) $1\frac{22}{37}$ (iii) $1\frac{54}{101}$ (iv) $1\frac{546}{1007}$

6. There are infinitely many answers.

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

7. $\frac{a - 2b}{a}$

8. $\frac{x-3}{2x-4}$

9. $\frac{5}{32a}$

10. $\frac{x}{3}$

11. $\frac{b+1}{b-1} 1$

12. 1

13. $\frac{4x+6}{4x+5}$

14. $\frac{13}{4}$

15. a. 7 b. -2

16. Combined, they painted $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$ of the room. Therefore, $\frac{4}{15}$ of the room was left to be painted.

It took Peter 1 hour, 40 minutes = 100 minutes to paint the remainder. Thus, $\frac{4}{5} = \frac{100}{x}$ yields that it

would have taken Peter 375 minutes, or $6\frac{1}{4}$ hours, to paint the room by himself.

17. The prime factorization of 285 reveals the three prime numbers: 3, 5, 19. Their sum is $3 + 5 + 19 = 27$. To verify this, we add $\frac{1}{3} + \frac{1}{5} + \frac{1}{19} = \frac{95}{285} + \frac{57}{285} + \frac{15}{285} = \frac{167}{285}$.
18. Point C is the midpoint of an 11-inch side of the paper. This means segment BC measures 5.5 inches. Since BC is folded onto DC , point D must be 5.5 inches from the left edge of the paper and $8.5 - 5.5 = 3$ inches from the right edge of the paper. Similarly, point G is 5.5 inches from the right edge and 3 inches from the left edge. Since points D and G are each 3 inches from an edge, they must be $8.5 - 6 = 2.5$ inches from each other. That is $2\frac{1}{2}$ inches as a mixed number.